# THE METHOD OF MONTHLY MEANS FOR DETERMINATION OF A SEASONAL VARIATION

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## 1. INTRODUCTION

Many of the mathematical processes used in statistical analysis are of such frequent application and are, relatively, so ancient in their origin that one ceases to think of the reasons for their well-known validity. When a new process is invented, however, it should be subjected to close scrutiny to determine the sphere in which its application is valid. The present paper deals with two methods which have been used for the determination of the seasonal variation in certain statistical data of economics. Arguments are presented against one method,[1] which is relatively complicated in its numerical details, and a complete logical foundation is given for a second method,[2] which involves merely the formation of monthly means.

Before considering the particular problems of this paper it is pertinent to decide on suitable criteria for testing the validity of a mathematical process used in statistical analysis. In the field of pure mathematics we may state certain definite hypotheses and, by logical reasoning, derive certain equally definite conclusions. It is realized, after very brief reflection, that most of the well-established mathematical methods of statistics have been taken over bodily from corresponding problems already solved in the field of pure mathematics. Such statistical methods have perfectly defined spheres of usefulness. We are usually able to state that if our statistical problem, when couched in mathematical terms, satisfies a given set of conditions, then our particular method leads us to conclusions, the accuracy of which is dependable. Many times, moreover, we are also able to state that even when the given set of conditions is not satisfied by our data, nevertheless the conclusions obtained by our method are the best, of a given type, that could possibly be obtained by *any* method.

To illustrate the preceding paragraph, consider the method of least squares for determining a line $y=at+b$ to fit a set of points in a plane containing the rectangular $(y, t)$ axes. This set of points may be considered as the graph of a time series, a table of the values of a variable quantity $y$ for a sequence of values of the time represented by $t$. The

[1] Persons, *Rev. of Economic Stat.*, Jan., 1919, p. 5.

[2] E. W. Kemmerer, *Seasonal Variation in the Demand for Money and Capital in the United States*, Report of the National Monetary Commission (1910).

following mathematical theorem is the logical basis for our statistical method in the present type of problem:

*If the set of points actually lies on a straight line, this is the line which is given by the method of least squares. Moreover, if the points do not lie on a straight line, that line which we obtain by our method is such that the sum of the squares of the distances (measured parallel to the $y$ axis) from the given points to our line is less than would be the case for any other straight line in the $(y, t)$ plane.*

The determination of the secular trend[1] in a time series may be based on this theorem, and our confidence in the results obtained is based upon the knowledge that the secular trend we determine is the best approximation we can hope for, where the word "best" is used in the sense of the method of least squares.

When a mathematical method of statistics is not based upon a theorem from pure mathematics, it is on the same plane as any empirical process. It should be subjected to experimental test in a controlled problem, where the results which should be obtained are known *a priori*. If a sensible problem can be proposed, where the results are known, in which the method in question leads to conclusions at variance with the truth, doubt should be cast upon all conclusions obtained by use of the method. Moreover, if two methods are available, of which one is founded logically and the other only empirically, with a slight doubt, perhaps, as to its general validity, it is obvious that the logically founded method is to be preferred. In the following discussion a logical background is given for the method of monthly means for determining seasonal variation. Afterwards an example of an admitted type is treated by Persons' method, and it is shown that the results obtained are widely at variance with the truth. The method of monthly means gives the correct result in this example.

## 2. THE METHOD OF MONTHLY MEANS

It is presumed that our data consist of a series of monthly entries of the values of some quantity over a series of $k$ consecutive years. It will be supposed that any secular change originally present in the data has been eliminated, for example, by fitting a straight line to the original data by the method of least squares and by then subtracting the ordinates of this straight line from the corresponding entries in the original data. Let $f(t)$ represent the value of our entry $t$ months from the zero date. For simplicity of nomenclature $t=0$ will be spoken of as January of the first year, $t=1$ as February of the first year, $t=12$ as January of the second year, $t=26$ as March of the third year, etc.

[1] Persons, *loc. cit.*, p. 12.

The method of monthly means for the determination of the seasonal variation in $f(t)$ is described in the following paragraph:

Form a new January entry by taking the arithmetic mean of all the January entries over the $k$ given years; form a February entry by taking the arithmetic mean of all February entries; . . . form a December entry similarly from the December entries over the $k$ years. This series of twelve monthly means is then to be taken as our best approximation to the values which $f(t)$ *would have had* in *all* years at the corresponding months if no causes had been influencing the values except those which lead to seasonal variation.

A seasonal variation is defined as a periodic change with the period one year. The graph of a seasonal variation over a period of one year is *not necessarily a single graceful wave* with a well-defined crest and trough. The essential feature of a strictly seasonal variation is that its numerical value is the same, at corresponding months, in all years. A seasonal variation thus yields the same values for the Januarys of all years, and similar constant results for the other months. Hence, it is obvious that the following theorem is true:

*Theorem* (1). *If $f(t)$ actually is a periodic function whose period is one year, the monthly entries obtained by our method are exactly the values of $f(t)$ at the corresponding months.*

The second theorem we shall state is designed to show explicitly the exact power of the method. Let $P(t)$ represent the periodic function, with the period one year, whose values for all Januarys, Februarys, etc., are the corresponding monthly means obtained above. Thus, $P(0)=P(12)=P(24)$, etc., = the January mean; $P(3)=P(15)=P(27)$, etc., = the April mean, etc.

*Theorem* (2). *Let $f(t)$ be any function of the time $t$ known from $t=0$ to $t=12k$, that is, over a period of $k$ years. Then, the sum of the squares of the residuals $[f(t)-P(t)]$, for all values of $t$, or*

(1) $[f(0)-P(0)]^2+[f(1)-P(1)]^2+ \; . \; . \; . \; +[f(12k-1)-P(12k-1)]^2$

*is smaller in value than it would be if any other periodic function with period one year were used in place of $P(t)$.*

It is seen that Theorem (2) shows $P(t)$ to be the answer to the following problem:

(2) *To determine a periodic function $P(t)$ which, in the sense of the method of least squares, is the best approximation to $f(t)$.*

The comparison of (2) with the theorem of the introduction shows the analogy between the method of monthly means and the method by which we determine a secular trend. The proof of Theorem (2) is extremely simple. It is well known that the sum of the squares of the deviations of a group of quantities from their arithmetic mean is less

than the sum of the squares of the deviations of the quantities from any number other than the arithmetic mean. The sum (1) consists of $12k$ terms which can be thought of as the sum of 12 groups of $k$ terms each, one group corresponding to each month of the year. The January group, for example, is $k$ times the sum of the squares of the deviations of the January entries about their arithmetic mean, which is $P(0)$. Hence, the January group in (1) is smaller in value than it would be if any other number were substituted in place of $P(0)$. Similar reasoning applies to the other eleven monthly groups in (1). Hence, the sum (1) is smaller in value than it would be if any other periodic function, with the period one year, were used in place of $P(t)$.

TABLE I

| Val. of $t$ | Val. of $f(t)$ | Val. of $t$ | Val. of $f(t)$ | Val. of $t$ | Val. of $f(t)$ | Val. of $t$ | Val. of $f(t)$ |
|---|---|---|---|---|---|---|---|
| 0 | 0.0 | 30 | −7.1 | 60 | 0.0 | 90 | 0.0 |
| 1 | 2.3 | 31 | −9.0 | 61 | 2.1 | 91 | −2.1 |
| 2 | 4.3 | 32 | −10.4 | 62 | 4.0 | 92 | −4.0 |
| 3 | 5.8 | 33 | −11.2 | 63 | 5.4 | 93 | −5.4 |
| 4 | 6.7 | 34 | −11.4 | 64 | 6.2 | 94 | −6.2 |
| 5 | 7.1 | 35 | −10.9 | 65 | 6.5 | 95 | −6.5 |
| 6 | 7.1 | 36 | −10.0 | 66 | 6.5 | 96 | −6.5 |
| 7 | 7.0 | 37 | −8.9 | 67 | 6.4 | 97 | −6.4 |
| 8 | 6.9 | 38 | −7.9 | 68 | 6.4 | 98 | −6.4 |
| 9 | 7.2 | 39 | −7.2 | 69 | 6.9 | 99 | −6.9 |
| 10 | 7.9 | 40 | −6.9 | 70 | 7.8 | 100 | −7.8 |
| 11 | 8.9 | 41 | −7.0 | 71 | 9.0 | 101 | −9.0 |
| 12 | 10.0 | 42 | −7.1 | 72 | 10.5 | 102 | −10.5 |
| 13 | 10.9 | 43 | −7.1 | 73 | 11.8 | 103 | −11.8 |
| 14 | 11.4 | 44 | −6.7 | 74 | 12.7 | 104 | −12.7 |
| 15 | 11.2 | 45 | −5.8 | 75 | 13.0 | 105 | −13.0 |
| 16 | 10.4 | 46 | −4.3 | 76 | 12.7 | 106 | −12.7 |
| 17 | 9.0 | 47 | −2.3 | 77 | 11.8 | 107 | −11.8 |
| 18 | 7.1 | 48 | 0.0 | 78 | 10.5 | 108 | −10.5 |
| 19 | 5.1 | 49 | 1.0 | 79 | 9.0 | 109 | −9.0 |
| 20 | 3.3 | 50 | 1.7 | 80 | 7.8 | 110 | −7.8 |
| 21 | 1.8 | 51 | 2.0 | 81 | 6.9 | 111 | −6.9 |
| 22 | 0.9 | 52 | 1.7 | 82 | 6.4 | 112 | −6.4 |
| 23 | 0.3 | 53 | 1.0 | 83 | 6.4 | 113 | −6.4 |
| 24 | 0.0 | 54 | 0.0 | 84 | 6.5 | 114 | −6.5 |
| 25 | −0.3 | 55 | −1.0 | 85 | 6.5 | 115 | −6.5 |
| 26 | −0.8 | 56 | −1.7 | 86 | 6.2 | 116 | −6.2 |
| 27 | −1.8 | 57 | −2.0 | 87 | 5.4 | 117 | −5.4 |
| 28 | −3.3 | 58 | −1.7 | 88 | 4.0 | 118 | −4.0 |
| 29 | −5.1 | 59 | −1.0 | 89 | 2.1 | 119 | −2.1 |

Let us apply the method of monthly means to an example arranged so as to forestall a certain objection which might be raised against the process. Let the values of our data $f(t)$ be as given in Table I, where $k=10$. The data have already been corrected for secular trend. The monthly means are found to be:

| Jan. | Feb. | March | April | May | June | July | Aug. | Sept. | Oct. | Nov. | Dec. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 1.0 | 1.7 | 2.0 | 1.7 | 1.0 | 0.0 | −1.0 | −1.7 | −2.0 | −1.7 | −1.0 |

In this case the function $P(t)$ which we determine as representing the seasonal variation has the monthly values listed in the table. The standard deviation of the residual function $f(t)-P(t)$ is found to be approximately 7. At first thought, one might consider this large value as an indication that our periodic function $P(t)$ was a poor approximation to the true seasonal variation. As a matter of fact, our function $P(t)$ in this case is *exactly equal* to the true seasonal variation, for $f(t)$ was computed from the following formulas:

$$f(t)=2\sin\left(\frac{t}{12}.360^\circ\right)+10\sin\left(\frac{t}{48}.360^\circ\right),\quad \textit{from } t=0 \textit{ to } t=48,$$
$$f(t)=2\sin(t\,.30^\circ),\quad \textit{from } t=48 \textit{ to } t=60,$$
$$f(t)=2\sin(t\,.30^\circ)+11\sin\left(\frac{t}{60}.360^\circ\right),\quad \textit{from } t=60 \textit{ to } t=120.$$

In other words, the $f(t)$ we are working with is the result of superimposing a varying long term oscillation on the seasonal variation given by $2\sin(t\,.30^\circ)$. Evidently, this problem shows that the size of the standard deviation of $f(t)-P(t)$ cannot be taken as a criterion of the applicability of the method.

The question naturally arises as to what is a proper criterion of applicability. The following theorem furnishes us with a theoretical answer:

*Theorem* (3). *The method of monthly means gives us the actual monthly values of the seasonal variation in case $f(t)$ is made up of the following component parts:*

(A) *A seasonal variation, strictly periodic throughout the period of years under consideration.*

(B) *A long term variation which consists of certain independent pieces, each extending over a whole number of years, where each piece represents a whole number of complete oscillations of a corresponding periodic function whose period is an integral number* (*two or greater*) *of years.*

(C) *A second, a third, etc., long-term variation having the characteristics specified in* (B).

The language of (A) and (B) is geometrical in nature, each component of $f(t)$ being thought of as a curve in the $(y, t)$ plane. It is desired that a certain implicit agreement be understood in the statement of Theorem (3). As a consequence of the theory of Fourier series, it is known that a periodic function with the period 5 years, for example, may have component parts with the respective periods $\frac{1}{2}$ of 5 years, $\frac{1}{3}$ of 5 years, $\frac{1}{4}$ of 5 years, $\frac{1}{5}$ of 5 years, or 1 year, $\frac{1}{6}$ of 5 years, etc. The agreement in our case is that the periodic functions referred to in (B) *have no component parts of period one year.* If the long-term varia-

tions originally had any one-year components, we are assuming that these were the same for all the variations and that this constant one-year oscillation was then thrown in under the variation mentioned in (A).

An example in which the conditions of Theorem (3) are satisfied would be given, for instance, by a function $f(t)$ defined for 17 years, consisting of

1. A seasonal variation.
2. A long-term variation which
   (a) for the first four years consists of two complete cycles of a periodic function with period two years,
   (b) for the next three years consists of one cycle of a periodic function whose period is three years,
   (c) for the next ten years consists of two complete cycles of a periodic function whose period is five years.
3. A second long-term variation which
   (a) for the first five years consists of one cycle of a periodic function whose period is five years,
   (b) for the next twelve years consists of four cycles of a periodic function whose period is three years.

A certain fundamental property of sines and cosines[1] makes Theorem (3) obvious to one familiar with the theory of Fourier series. Consider the particular function cos $(t\ .15°)$, which has two years as its period of oscillation. Let $h$ be *any* positive integer and let $A$ be less than $720°/h$. Then the property referred to is the fact that

$$(3)\ \cos A+\cos(A+b)+\cos(A+2b)+\quad .\ \ .\ \ .\quad +\cos[A+(h-1)b]=0,$$

where $b=720°/h$. In the averaging process of the method of monthly means we meet consequences of this property which is possessed by all sines and cosines. In determining the mean for any one month, we form a sum of the values of $f(t)$, which, for February, would be

$$(4)\ f(1)+f(1+12)+\quad .\ \ .\ \ .\quad +f[1+(k-1)12].$$

The effect of the long-term variations (B) and (C) on the sum in (4) is zero, because of equations like (3) which would hold for each piece of (B). The seasonal variation, however, furnishes to (4) exactly $k$ times the value the variation takes on at February in all years. Hence, the monthly mean of $f(t)$ for February is the exact value of the seasonal variation for this month. Similar reasoning applies in the case of the other monthly means, so that Theorem (3) is true.

The statements of Theorems (2) and (3) justify us in applying the method of monthly means in the analysis of data which are affected

[1] Bôcher, *Annals of Mathematics*, Second Series, Vol. 7 (1906), p. 135, Formula (63).

by the business cycle. In such cases, it is true, the assumptions of Theorem (3) are not exactly satisfied. Nevertheless, there is enough similarity between the conditions of Theorem (3) and the actual conditions affecting the economic data to give us confidence in the results obtained by taking monthly means. Moreover, we always have the unquestionable truth of Theorem (2) as a foundation for our belief. The ultimate test of any method in statistics is, of course, the degree of success it attains in actual practice. This test has not as yet been applied to any great extent in the case of the present method, but it is hoped that its simplicity and ease of application will permit such tests in the future. One application to a set of data is treated in the next part of this paper in connection with a comparison of the method used by Persons and that of monthly means.

### 3. COMPARISON OF THE METHOD OF MONTHLY MEANS WITH THAT OF PROFESSOR PERSONS

Consider the data given in Table II. This set of values was computed by means of the following formulas:

$$f(t) = 15 + \sin t(30°) + 4 \sin t(10°), \quad \textit{from } t = 0 \textit{ to } t = 36,$$
$$f(t) = 15 + \sin t(30°) + 6 \sin t(10°), \quad \textit{from } t = 36 \textit{ to } t = 72,$$
$$f(t) = 15 + \sin t(30°) + 2 \sin t(10°), \quad \textit{from } t = 72 \textit{ to } t = 108.$$

We have in this case data over 9 years which are the result of compounding the seasonal variation $15 + \sin t(30°)$ with the long-term variations given, respectively, by the expressions $4 \sin t(10°)$, $6 \sin t(10°)$, and $2 \sin t(10°)$. The conditions of Theorem (3) of the previous section are satisfied, and, therefore, the method of monthly means leads us to the exact values of the seasonal variation, which are given in the following table:

| Jan. | Feb. | March | April | May | June | July | Aug. | Sept. | Oct. | Nov. | Dec. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 15.00 | 15.50 | 15.87 | 16.00 | 15.87 | 15.50 | 15.00 | 14.50 | 14.13 | 14.00 | 14.13 | 14.50 |

Let us consider obtaining the values of the seasonal variation by Persons' method. First, we are required to compute the ratios of each entry in the $f(t)$ table to the preceding entry. The results of this computation are arranged in monthly columns in Table III. Then, in the resulting series of relatives for each of the twelve months, we pick out the median values, which are listed in Table IV. In using the method under consideration, the assumption now is made that the seasonal variation causes, in the average, each February

value to be 1.018 (Table IV) times the preceding January value, each March value to be 1.016 times the preceding February value, etc. It is not our present purpose to consider the non-mathematical arguments in justification of this assumption. However, it must be admitted that the assumption appears to be sensible and would be very acceptable if no logically-founded method were available and if the results to be found below were not on hand as contrary evidence. Instead of considering an argument, let us compare the results of this method with the actual values of the seasonal variation which should be obtained in the present instance.

TABLE II

TABULATION OF $f(t)$ BY MONTHS

| Year | Jan. | Feb. | March | April | May | June | July | Aug. | Sept. | Oct. | Nov. | Dec. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1st | 15.000 | 16.196 | 17.234 | 18.000 | 18.438 | 18.564 | 18.464 | 18.260 | 18.074 | 18.000 | 18.074 | 18.260 |
| 2d | 18.464 | 18.564 | 18.438 | 18.000 | 17.234 | 16.196 | 15.000 | 13.804 | 12.766 | 12.000 | 11.562 | 11.436 |
| 3d | 11.536 | 11.740 | 11.926 | 12.000 | 11.926 | 11.740 | 11.536 | 11.436 | 11.562 | 12.000 | 12.766 | 13.804 |
| 4th | 15.000 | 16.544 | 17.918 | 19.000 | 19.724 | 20.096 | 20.196 | 20.140 | 20.044 | 20.000 | 20.044 | 20.140 |
| 5th | 20.196 | 20.096 | 19.724 | 19.000 | 17.918 | 16.544 | 15.000 | 13.456 | 12.082 | 11.000 | 10.276 | 9.904 |
| 6th | 9.804 | 9.860 | 9.956 | 10.000 | 9.956 | 9.860 | 9.804 | 9.904 | 10.276 | 11.000 | 12.082 | 13.456 |
| 7th | 15.000 | 15.848 | 16.550 | 17.000 | 17.152 | 17.032 | 16.732 | 16.380 | 16.104 | 16.000 | 16.104 | 16.380 |
| 8th | 16.732 | 17.032 | 17.152 | 17.000 | 16.550 | 15.848 | 15.000 | 14.152 | 13.450 | 13.000 | 12.848 | 12.968 |
| 9th | 13.268 | 13.620 | 13.896 | 14.000 | 13.896 | 13.620 | 13.268 | 12.968 | 12.848 | 13.000 | 13.450 | 14.152 |

TABLE III

TABULATION OF RELATIVES BY MONTHS

| Year | Jan. | Feb. | March | April | May | June | July | Aug. | Sept. | Oct. | Nov. | Dec. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1st | 1.060[1] | 1.800 | 1.064 | 1.045 | 1.024 | 1.006 | .995 | .989 | .990 | .996 | 1.004 | 1.010 |
| 2d | 1.011 | 1.005 | .994 | .976 | .957 | .940 | .926 | .920 | .925 | .940 | .963 | .990 |
| 3d | 1.009 | 1.017 | 1.016 | 1.006 | .994 | .984 | .983 | .991 | 1.010 | 1.038 | 1.064 | 1.081 |
| 4th | 1.087 | 1.103 | 1.083 | 1.060 | 1.038 | 1.019 | 1.005 | .997 | .995 | .998 | 1.002 | 1.005 |
| 5th | 1.003 | .995 | .981 | .963 | .943 | .923 | .907 | .897 | .897 | .910 | .935 | .963 |
| 6th | .990 | 1.006 | 1.010 | 1.004 | .996 | .990 | .994 | 1.010 | 1.038 | 1.070 | 1.098 | 1.114 |
| 7th | 1.114 | 1.057 | 1.044 | 1.027 | 1.009 | .993 | .982 | .979 | .983 | .994 | 1.006 | 1.017 |
| 8th | 1.021 | 1.018 | 1.007 | .991 | .974 | .958 | .946 | .943 | .951 | .966 | .989 | 1.009 |
| 9th | 1.023 | 1.026 | 1.021 | 1.007 | .993 | .980 | .974 | .977 | .991 | 1.011 | 1.035 | 1.052 |

[1] January, 10th year.

From the monthly medians of the link relatives listed in Table IV we obtain[1] a corresponding series of relatives on January as a base. For example, the March entry is found by computing (1.018) (1.016) = 1.034, the April value is (1.006) (1.034) = 1.040, etc., as given in Table IV. By this method we obtain for January itself the value (1.021) (.978) = .999, instead of 1.000. This discrepancy[1] is spread over the whole year, giving the final adjusted values found in Table IV. For purposes of comparison the true values of the monthly relatives of the seasonal variation have been computed on January as a base and are listed in Table IV.

[1] Persons, *loc. cit.*, p. 31.

To find actual numerical values of the seasonal variation from the final adjusted relatives obtained by Persons' method, we compute the mean of all the entries in the original Table II, which is found to be 15. Then we multiply in succession by the monthly relatives. The results of this computation are given in Table IV, with the true values of the seasonal variation listed simultaneously for purposes of comparison. The true monthly increments, or decrements, of the seasonal variation and those of the computed variation are likewise given in Table IV.

TABLE IV

| Month | Medians of relatives | Medians on January as base | Medians adjusted | Actual monthly relatives on January as base | Persons' seasonal variation | Actual seasonal variation | Persons' monthly increment | Actual monthly increment |
|---|---|---|---|---|---|---|---|---|
| January...... | 1.021 | .999 | 1.000 | 1.000 | 15.00 | 15.00 | .31 | .50 |
| February..... | 1.018 | 1.018 | 1.018 | 1.033 | 15.27 | 15.50 | .27 | .50 |
| March....... | 1.016 | 1.034 | 1.034 | 1.058 | 15.52 | 15.87 | .25 | .37 |
| April........ | 1.006 | 1.040 | 1.041 | 1.067 | 15.61 | 16.00 | .09 | .13 |
| May......... | .994 | 1.034 | 1.035 | 1.058 | 15.52 | 15.87 | −.09 | −.13 |
| June......... | .984 | 1.018 | 1.018 | 1.033 | 15.27 | 15.50 | −.25 | −.37 |
| July......... | .982 | .999 | 1.000 | 1.000 | 15.00 | 15.00 | −.27 | −.50 |
| August...... | .979 | .978 | .979 | .967 | 14.69 | 14.59 | −.31 | −.50 |
| September.... | .990 | .969 | .969 | .942 | 14.54 | 14.13 | −.15 | −.37 |
| October...... | .996 | .965 | .966 | .933 | 14.48 | 14.00 | −.06 | −.13 |
| November.... | 1.004 | .969 | .970 | .942 | 14.53 | 14.13 | .05 | .13 |
| December.... | 1.010 | .978 | .979 | .967 | 14.69 | 14.50 | .16 | .37 |

The conclusions to be drawn from the above problem are obvious. By Persons' method we are led through the computation of 108 relatives, the determination of twelve medians, the reduction of these medians to January as a base, the adjustment of these final values to distribute a certain discrepancy over the whole year, and then to the transfer from these adjusted values to actual numerical values of the seasonal variation. Finally, we discover that the error in the monthly decrements or increments is generally about 50 per cent. On the other hand, the method of monthly means leads to the correct numerical values of the seasonal variation after the easy computation of twelve monthly means.

The actual statistical series in which we search for a seasonal variation may differ from the simple case we have treated above, but they differ in that they are much more complicated. If Persons' method does not yield the correct results in a simple case, we must logically consider that its applicability in a complicated instance is open to question.